Minitab®

SPC

Quality Methods: Control Charts for Individual Units

Previous lessons/homework considered subgroups: $\bar{x}$'s

Dr. Diane Evans

ROSE-HULMAN
INSTITUTE OF TECHNOLOGY

Control Charts for Individuals (I or X) and Moving Ranges (MR)

- In some situations (cost, availability) and processes (destructive testing), the sample or subgroup size is 1.
- How do we apply! $\bar{x}$ charts? – ***Not Possible***
- Use I (individuals) and MR (moving range, used to measure variability or process spread) charts.
- How can we tell if Individuals are normally distributed (in order to justify the assumption of normality for the control chart)?
- How can we tell if consecutive points are independent of each other?

Assumptions: When constructing I-MR charts, the data must:

1. Come from a **normally distributed** process

2. Be **independent**
If **data is not from a normal distribution**, then

(a) We can still plot it on an *I-MR* chart, but the Type I "out of control" rules are invalid; just look for patterns or bad behavior in the chart

(b) Transform the data into normally distributed data and plot the transformed data on *I-MR* charts

If the **data is not independent**, then

(a) The UCL and LCL may be too tight and cause possible false alarms (e.g., clustering from batches). **FDA story from Dr. Franklin. Toyota example from ELSS conference**.

(b) The UCL and LCL may be to wide and cause us miss a given process that is out of control.

(c) When plotting data on *I-MR* charts, the lack of independence may show up in the chart on its own (e.g., trends), or you may not be able to identify it at all.

I-MR Charts: Facts and Warnings

1. To calculate *MR* – take two consecutive *X* values, calculate:

$$MR_i = \left| X_i - X_{i-1} \right|$$

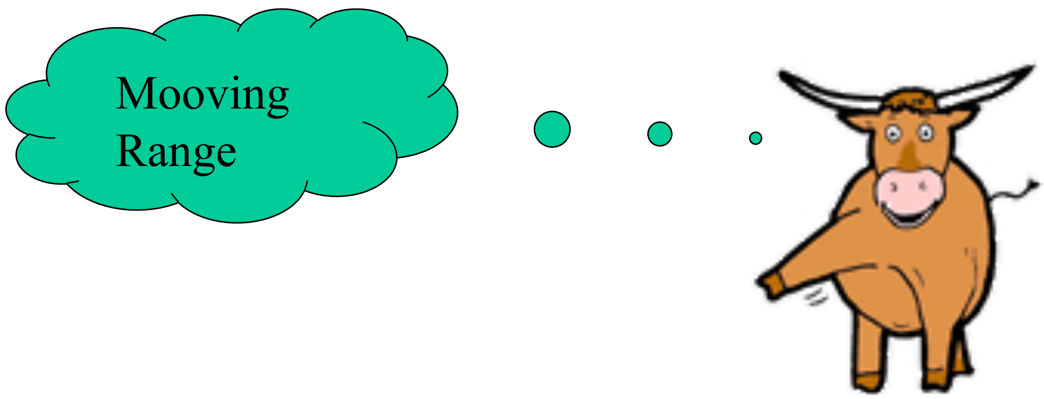
2. **Moving ranges** are **correlated** because two consecutive moving ranges share an identical data point

3. If there is **special cause variation** between two consecutive individual points – you'll get an "inflated" moving range estimate.

4. If there is **"clumping"** between two or more consecutive points, you'll get a "deflated" moving range estimate.

Sample *I* and *MR* data:

Index	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Waiting Times in ARA lunch line	8.4	6.5	10.8	9.7	9.0	9.4	10.2	8.1
MR	*	1.9	4.3	1.1	0.7	0.4	0.8	2.1



Estimate of the process standard deviation σ

$$= \overline{MR} / d_2$$

where $\overline{MR}$ is the average of the moving ranges in **successive order**. Use ***n* = 2 to compute *d*$_2$** as long as two consecutive points are used to compute each *MR*.

- Note that if we have a total of *k* individual data points, then we will have *k* – 1 moving ranges.
- As with the other charts ($\bar{x}$ and *R*, $\bar{x}$ and s), we have *IMR* charts **given current process data** or when **standards are given** – that is, we are given μ_0 and σ_0

I and *MR* Control Limits Based on *Data*; No Standards Given

- *I* Chart Limits

$$UCL_X = \overline{X} + 3\overline{MR} / d_2$$

$$LCL_X = \overline{X} - 3\overline{MR} / d_2$$

- *MR* Chart Limits

$$UCL_{MR} = D_4 MR$$

$$LCL_{MR} = D_3 \overline{MR}$$

I and *MR* Control Limits for Given Targets

We are given $\bar{x}_0$ (or μ_0) and σ_0

- *I Chart Limits*

$$UCL_X = \overline{X}_0 + 3\sigma_0$$

$$LCL_X = \overline{X}_0 - 3\sigma_0$$

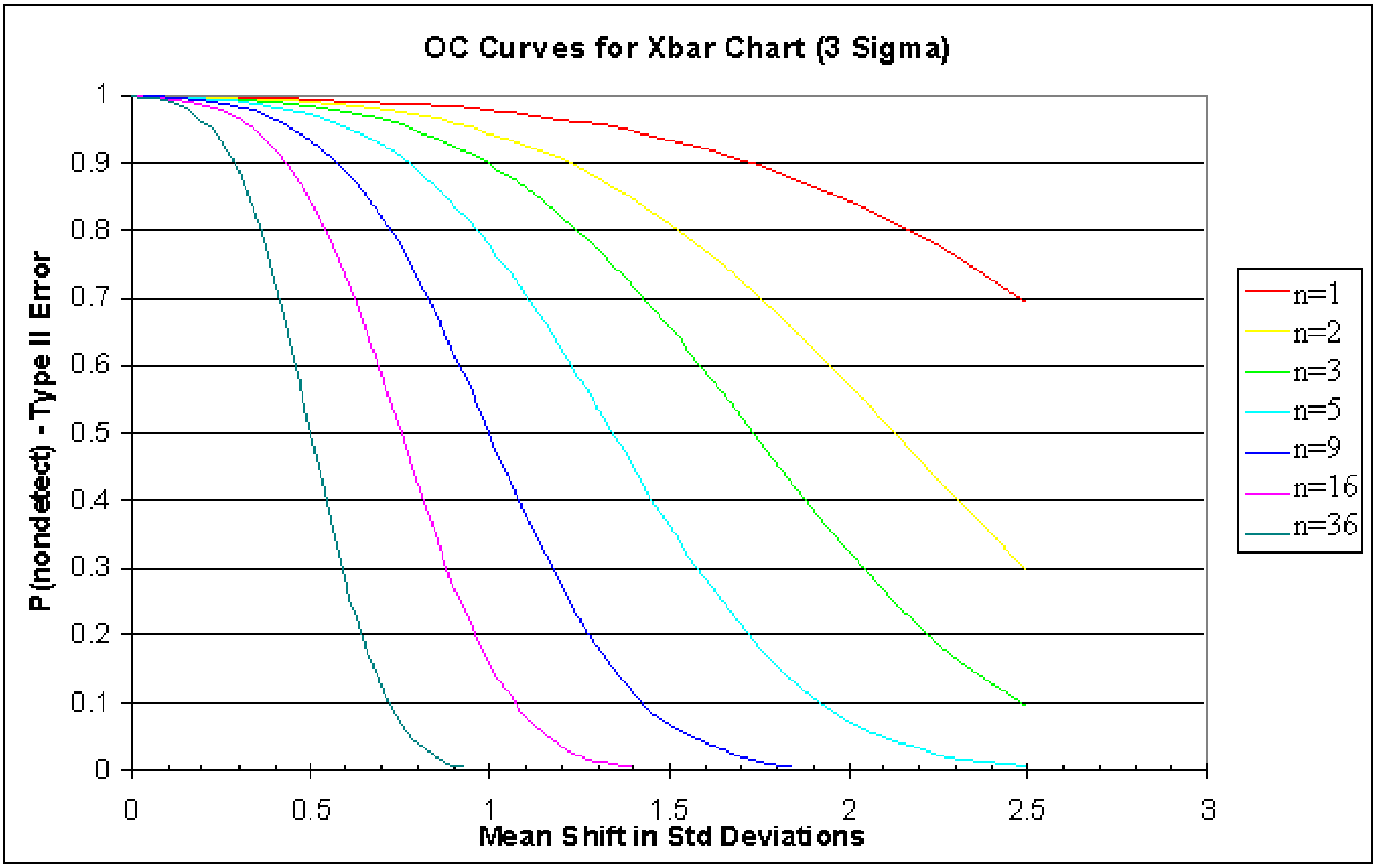
- ***MR* Chart Limits,** where we assume $n = 2$ for d_2 if two consecutive points are mused for the moving range

$$CL_{MR} = d_2 \sigma_0$$

$$UCL_{MR} = D_4 d_2 \sigma_0$$

$$LCL_{MR} = D_3 d_2 \sigma_0$$

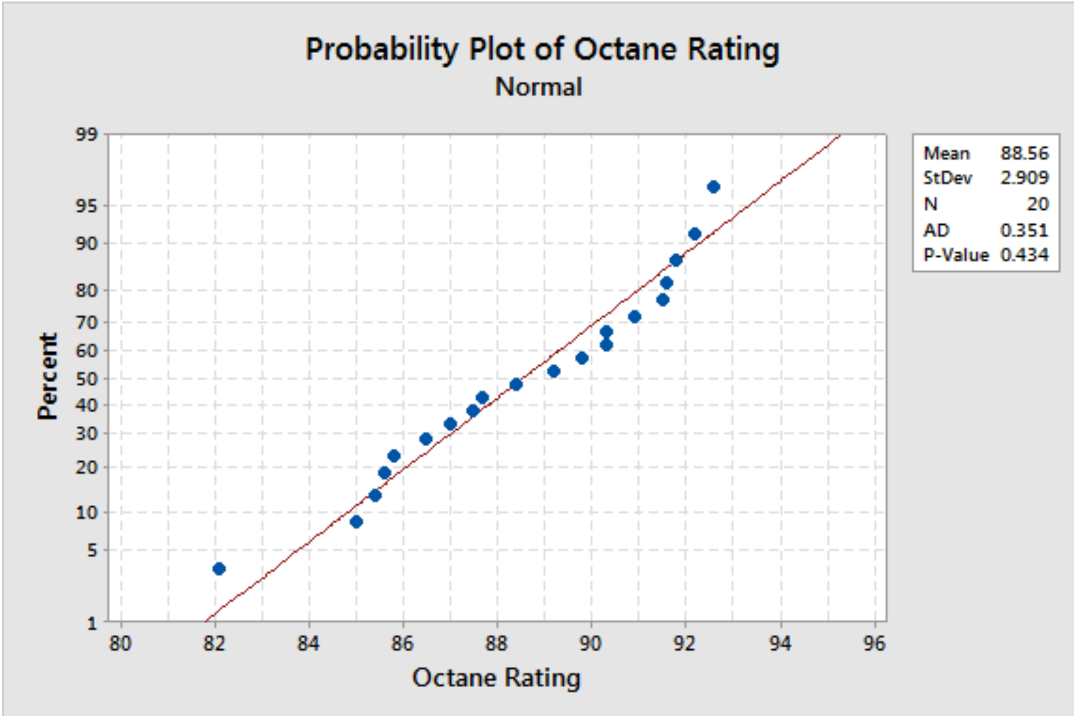
I chart not as sensitive to mean shifts as $\bar{x}$ chart – takes longer to detect shift on *I* chart. Recall *OC* Curve Summary for increasing values of *n*



Example 1. In a gasoline-blending plant, the quality of the output as indicated by its octane rating is measured for a sample taken from each batch. The observations from 20 such samples are in the file Lesson12DATA_IMR_GASOCTANE. Construct *I-MR* charts, where the moving range is constructed based on two consecutive observations.

Minitab: Stat > **Control Charts** > **Variables for Individuals** > ***I/MR***; I/MR Option: Moving range calculated from sample size of 2 (i.e., 2 consecutive observations)

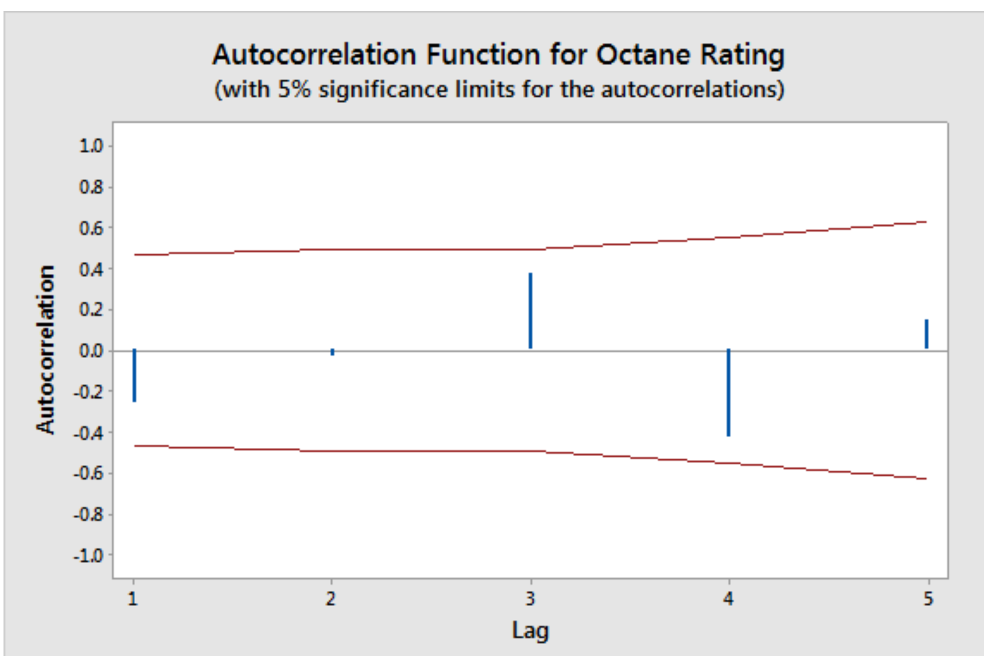
Check **Normality Assumption:**



Check **Autocorrelation** or dependency:

Minitab:

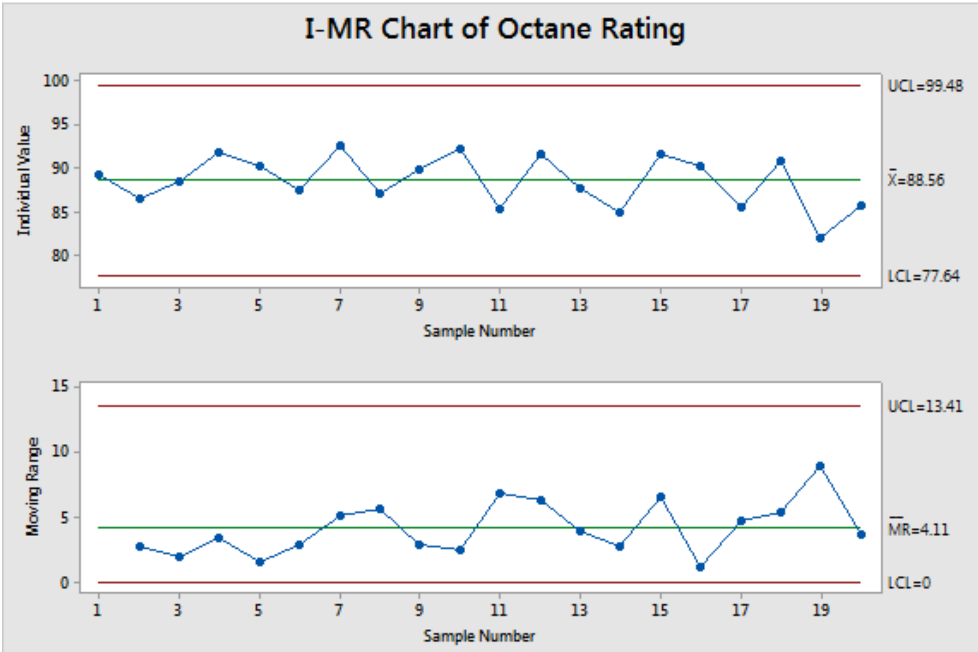
Stat > **Time Series** > **Autocorrelation**



Minitab:

Stat > **Control Charts** > **Variables Charts for Individuals** > ***I/MR***

I/MR Option: Length of moving range: 2

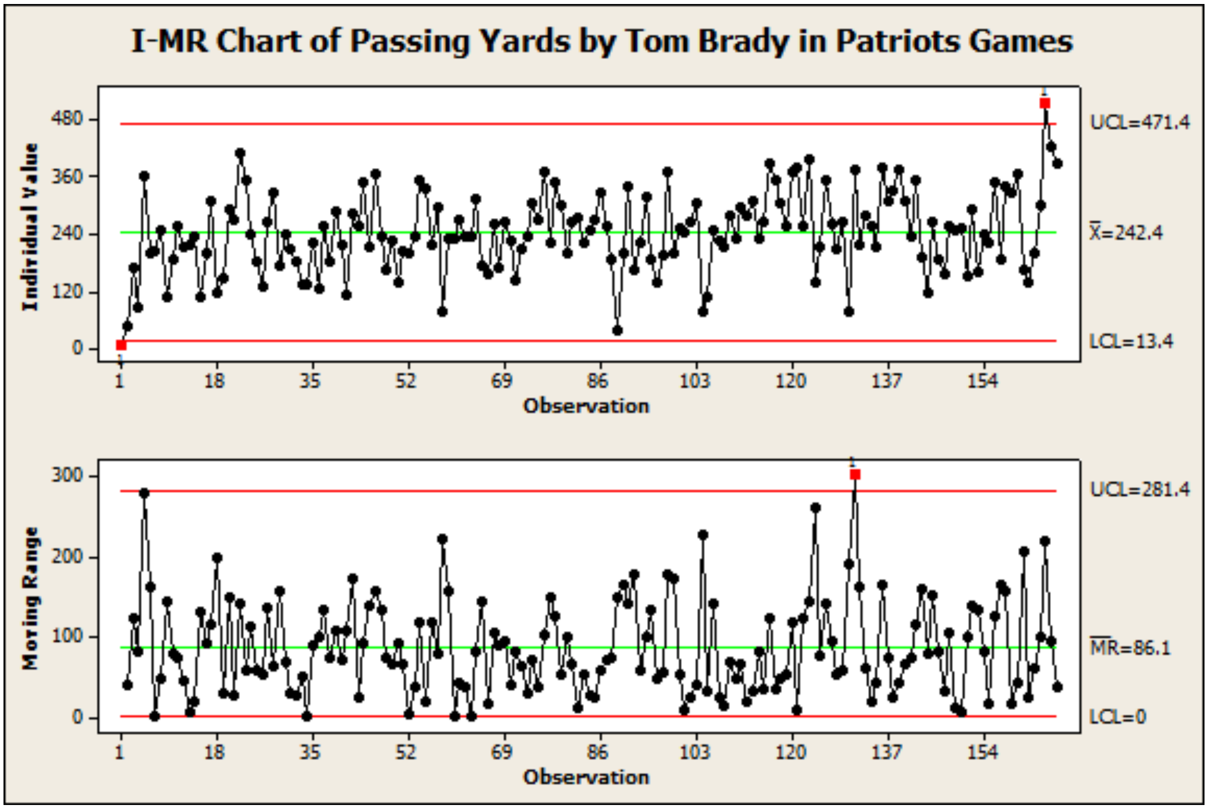


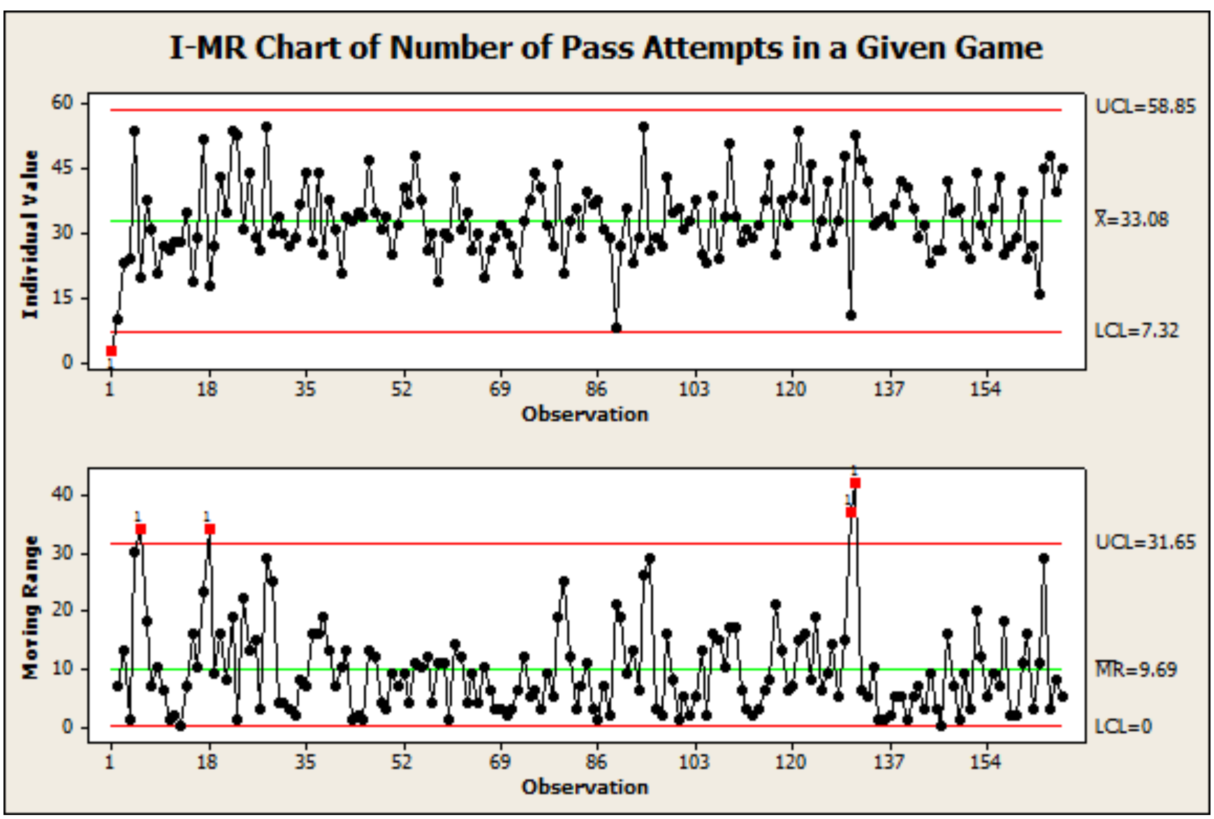
Tom Brady and Control Charts

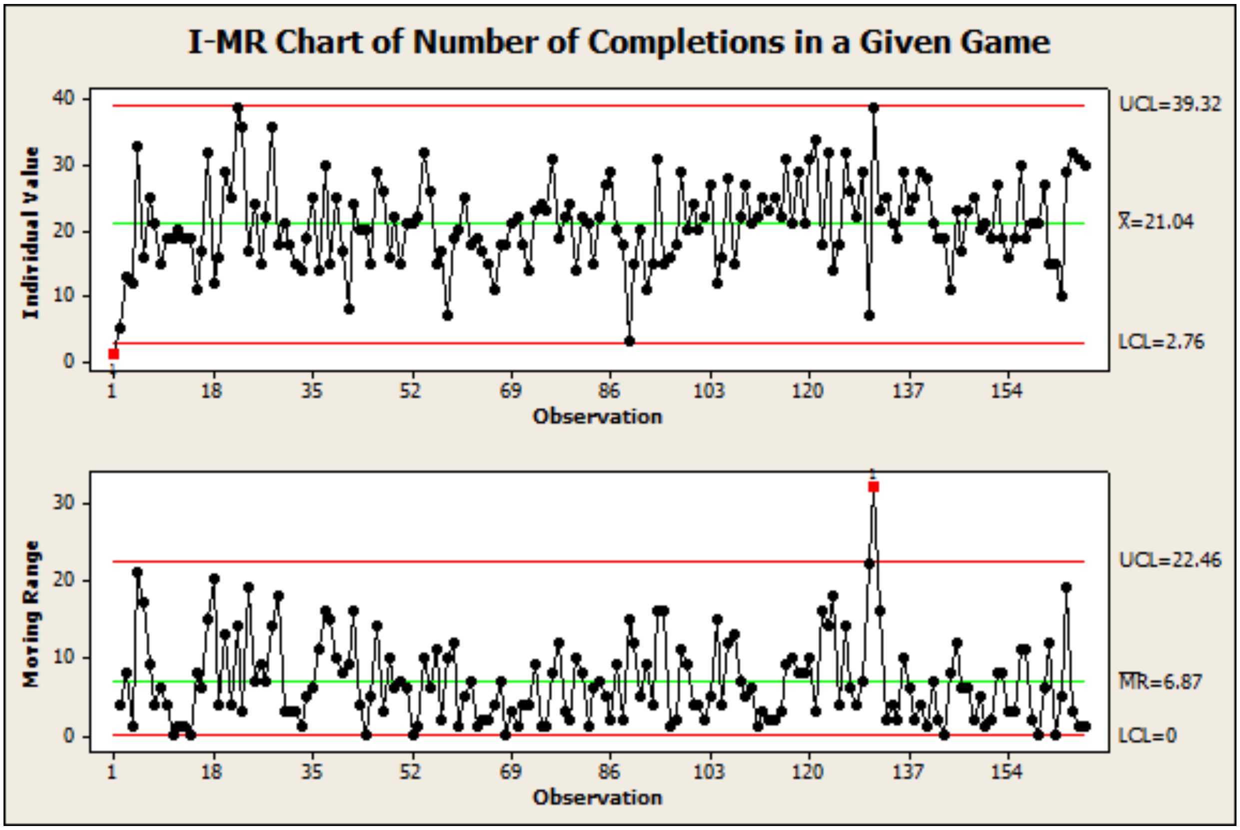
by Ben Jones, 2011 September 27

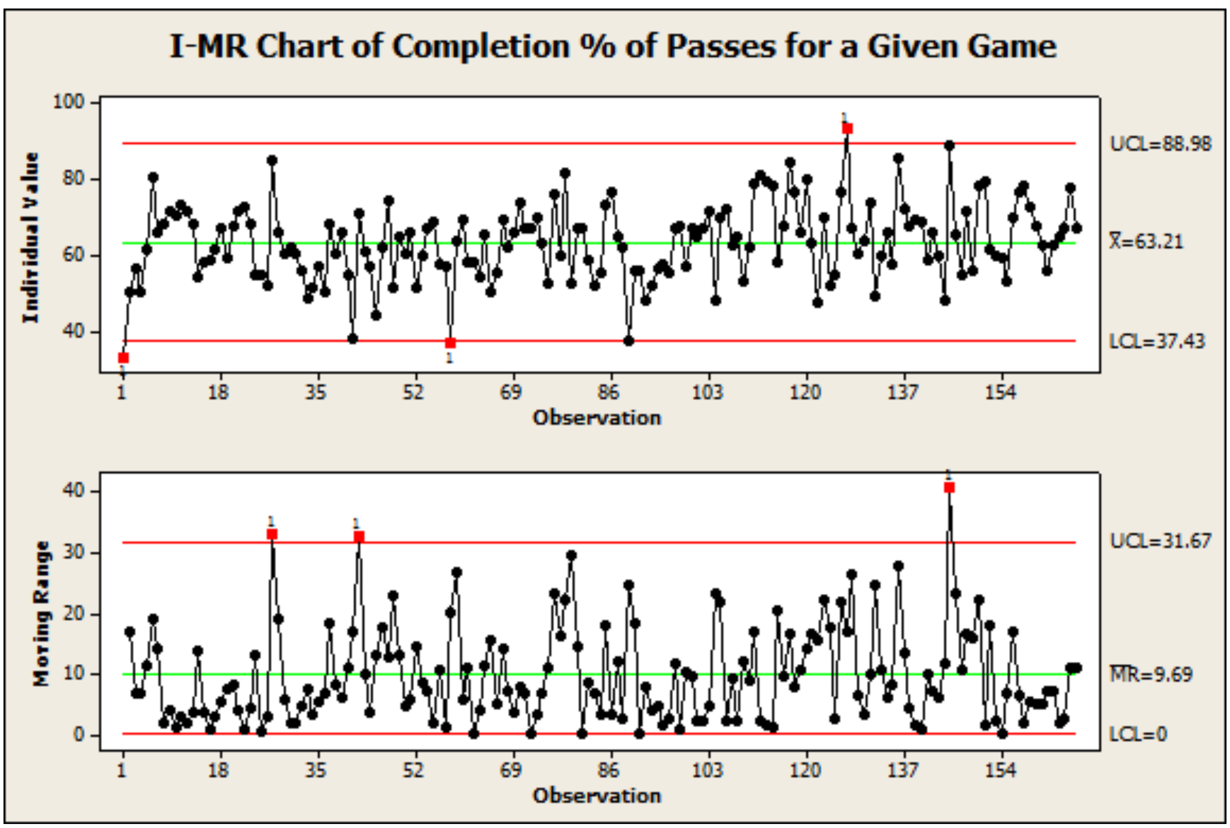
- In September 2011, Tom **Brady** of the New England Patriots NFL team threw for **over 500 yards** in a single **game** for the first time in his career.
- His 517 yard tally against the Miami Dolphins on 9/12/2011 was over ***100 yards more*** than his ***previous career best: 410 yard*** performance 9/22/2002
- But was this a **statistically significant event**, and if so, to what can we attribute it (i.e., special cause variation)?
  - Completed a higher percentage of passes?
  - Maybe he threw more passes than usual?
  - His **girlfriend** agreed to marry him?

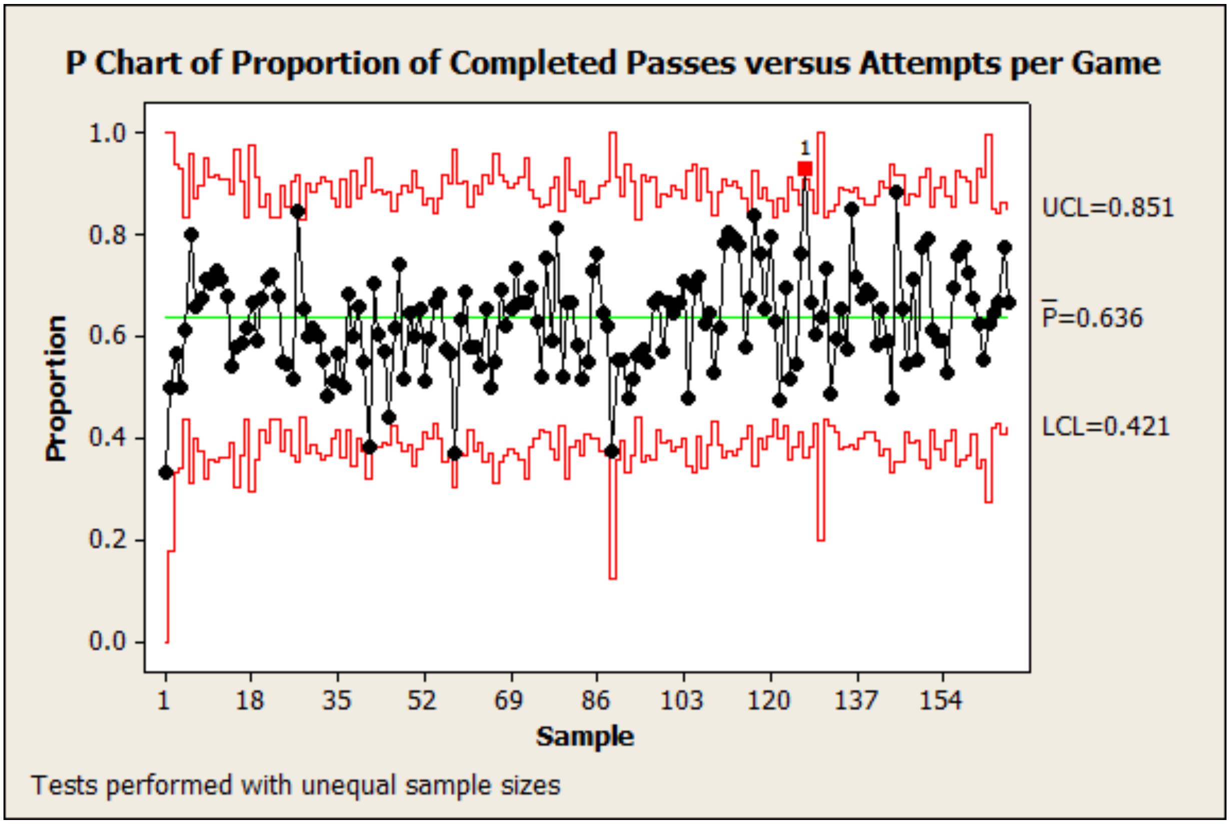
517 yards is an "outlier" on the *I* chart; it's above the UCL

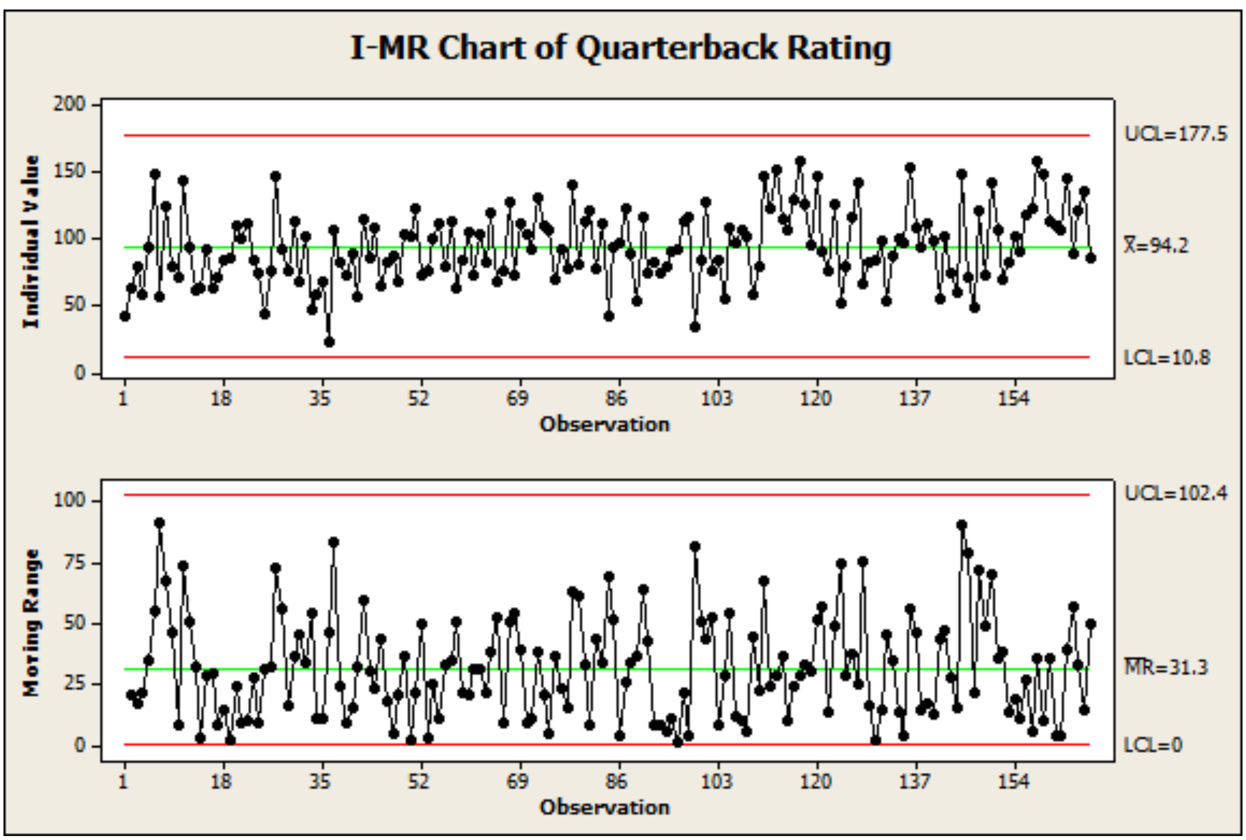


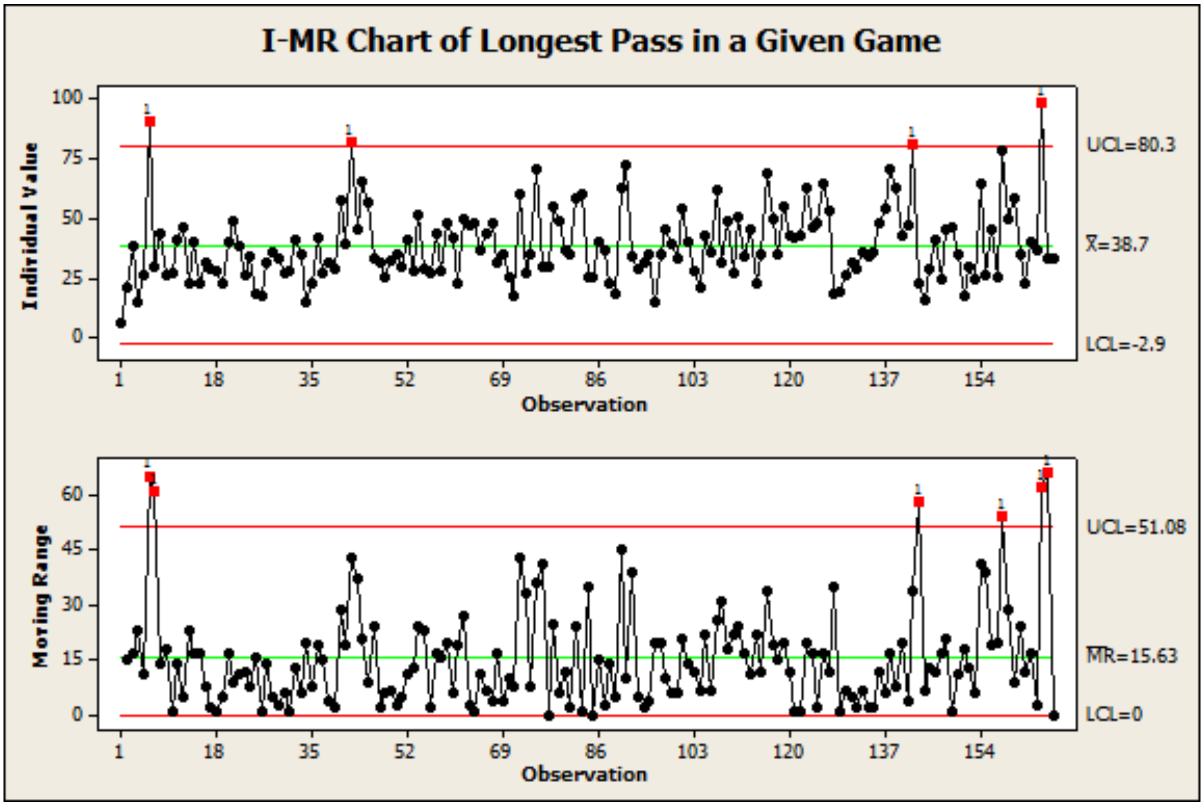



I-MR Chart of Number of Completions in a Given Game
Individual Value
UCL=39.32
X̄=21.04
LCL=2.76
Observation
Moving Range
UCL=22.46
M̄R=6.87
LCL=0
Observation

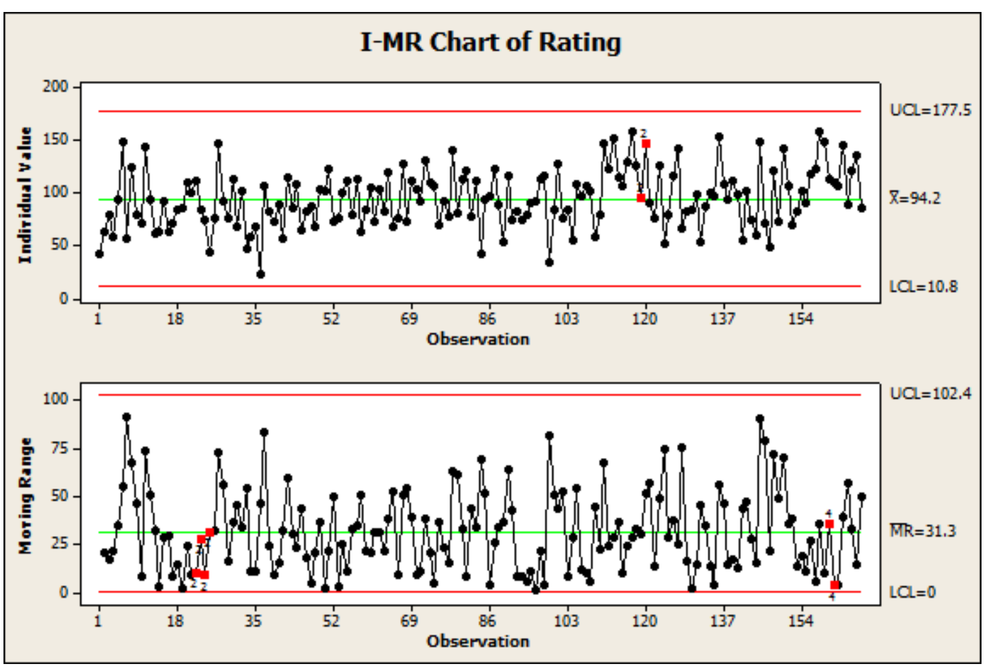

I-MR Chart of Completion % of Passes for a Given Game
Individual Value
UCL=88.98
X̄=63.21
LCL=37.43
Observation
Moving Range
UCL=31.67
M̄R=9.69
LCL=0
Observation


P Chart of Proportion of Completed Passes versus Attempts per Game
Proportion
1.0
0.8
0.6
0.4
0.2
0.0
1
UCL=0.851
P̄=0.636
LCL=0.421
1
18
35
52
69
86
103
120
137
154
Sample
Tests performed with unequal sample sizes


I-MR Chart of Quarterback Rating
Individual Value
200
150
100
50
0
UCL=177.5
X̄=94.2
LCL=10.8
1
18
35
52
69
86
103
120
137
154
Observation
Moving Range
100
75
50
25
0
UCL=102.4
M̄R=31.3
LCL=0
1
18
35
52
69
86
103
120
137
154
Observation



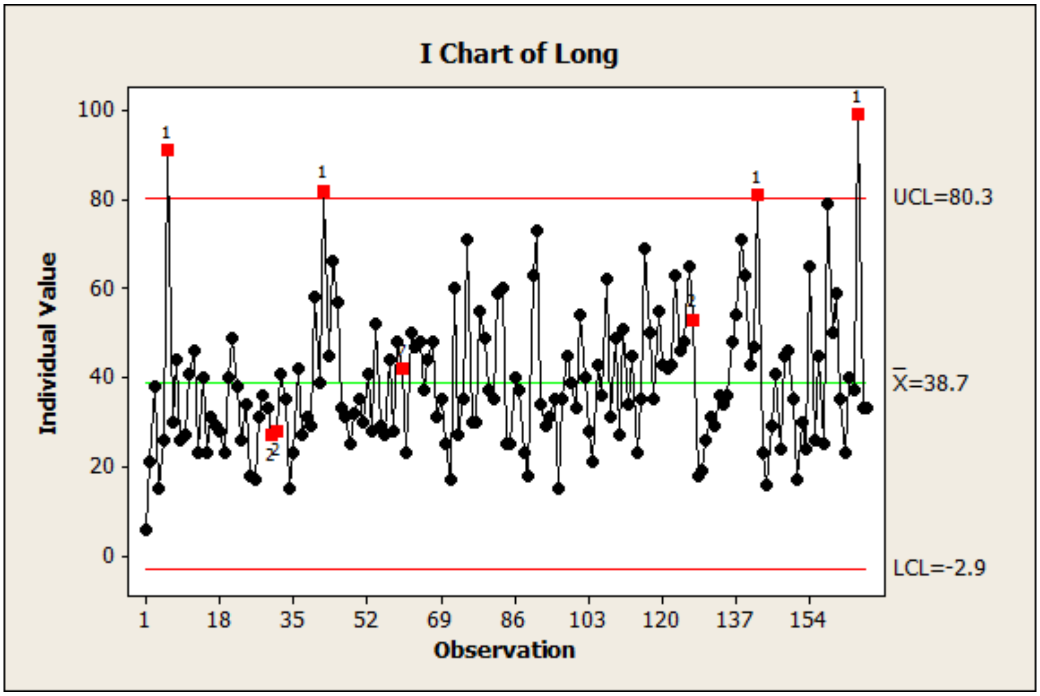
- 9/12/2011 Patriots game scenario: In the 4th quarter, with less than 6 minutes left, **Brady threw a 99 yard touchdown** strike to Wes Welker, racking up the most yards possible for a single play from scrimmage.
- All of the **other Brady statistics are in control** on this date – meaning Brady's performance was not abnormal (for Brady) in respect to attempts, completions, completion percentage, average yards per pass, or even quarterback rating
- Something else to consider – **"out of control" points due to the "other rules."** On several of these charts, I'm going to run all 8 "out of control" rules



Test Results for MR Chart of Rating

TEST 2. 9 points in a row on same side of center line.
Test Failed at points: 22, 23, 24, 25

TEST 4. 14 points in a row alternating up and down.
Test Failed at points: 160, 161



What if **dependency DOES exist** in the Individuals data?

Two different general solutions to the problem emerge in the literature.
(Vanhatalo and Murat Kulahci, 2014)

1. Adjust the control limits of the traditional charts, for example, by accounting for the autocorrelation in the estimation of the process standard deviation.

2. Fit a time series model to the data and then apply the traditional control charts to the residuals from the model—sometimes referred to as the 'Alwan and Roberts method.' *

While a residuals chart has a higher probability in detecting a shift in the process mean in the first plotted point after the shift occurs, the detection ability at future points depends on the autocorrelation structure potentially liable to cause excessive delays in detecting an out-of-control signal.

The Effect of Autocorrelation on the Hotelling T^2 Control Chart

Erik Vanhatalo and Murat Kulahci
Article first published online: 28 SEP 2014

*Alwan LC, Roberts HV. Time-series modeling for statistical process control. *Journal of Business & Economic Statistics* 1988; 6(1): 87–95